Surface Area of a Wine Glass

Professor Jerison found the volume of a "wine glass" shape formed by revolving the graph of $y = e^x$ $(0 \leq x \leq 1)$ about the y-axis. Set up but do not evaluate an integral to compute the surface area of that shape.

1/9/25

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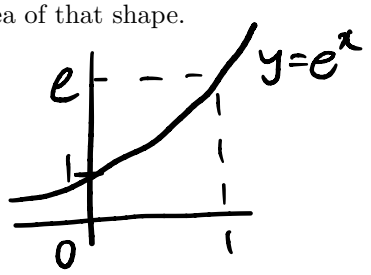
$V = \pi(e-2)$



$dA = 2\pi x\, ds$

$= 2\pi \cdot \ln y \cdot \sqrt{1+\frac{1}{y^2}}\, dy$

$\therefore A = S = 2\pi \int_1^e \ln y \sqrt{1+\frac{1}{y^2}}\, dy$

$S = 2\pi \int_0^1 x\sqrt{1+e^{2x}}\, dx$

$ds^2 = dx^2 + dy^2$

$\Rightarrow ds = \sqrt{dx^2+dy^2}$

$\frac{ds}{dy} = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}$

$\Rightarrow S = \int \sqrt{1+x'^2}\, dy$

$\Rightarrow ds = \sqrt{1+x'^2}\, dy$

$y = e^x \Rightarrow x = \ln y$

$\Rightarrow x' = \frac{1}{y}, \; x'^2 = \frac{1}{y^2}$

$ds = \sqrt{1+\frac{1}{y^2}}\, dy$